Oral-Example-Geometry-Topology

All-round

1. (a) Let $f : \mathbb{CP}^2 \to \mathbb{CP}^2$ be a continuous map of degree d with $|d| \geq 2$. Show that there are at least 3 fixed points.

   (b) Show that $\mathbb{CP}^{2n}$ does not cover any manifold except itself.

2. Let D be a bounded and simply connected domain in $\mathbb{R}^2$, and Γ be the boundary of D. Set
$$A := Area(D), \quad L := Length(\Gamma).$$

   (a) Prove that
$$4\pi A \leq L^2,$$
   and the equality holds iff Γ is a circle.

   (b) Generalize the theorem above to the case where D is a compact and simply connected minimal surface with boundary in $\mathbb{R}^3$.